A theoretical Study on Temperature Distribution of circular Saw-blade

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Introductory Remarks

It was formerly reported[1] by H.SUGIHARA that the greatest obstacle in sawing with circular saw machine is the buckling of saw-blade and that it is caused only by the compression stress depending on the state of temperature distribution in it.

One of the key-points for solving this problem is to know accurately the temperature distribution, but it is not easy due to the difficulty of measuring the temperature of saw-blade in the state of sawing in high speed revolution. There are few results[2),3)] of experiments measuring it, but these are not satisfactory.

In this paper the authors, with the object of contributing something to solve this problem, introduce a differential equation to express the temperature distribution, solve it actually under rough assumptions and consider it comparing with the experimental results.

Composition of the differential equation

The temperature rising and gradient in the blade are caused by the heat generated in the outer-most region from the cutting work and friction in sawing, and the heat generated from the side friction of blade. Moreover the saw-blade runs so much longer in the air than in the wood that the dispersion of heat into the air from the side surfaces of blade have to be taken into consideration as a factor influencing on the temperature gradient.

It may be assumed that these three factors — the heat generated in the outermost region, the one from the side friction and the dispersion of heat from the surface of blade— determine the temperature distribution in the sawblade.

Then we choose the centre of circular sawblade as the origin of polar coordinates (r, θ), r_a the radius of the circle of tooth-bottom, r_b the radius of the flange, and h the thickness of blade. (Fig. 1).

Strictly speaking, these three factors should be represented as a function of r, θ and

t respectively, but it may be assumed that the speed of revolution of saw-blade is so much larger than the feed speed of wood that the heat is uniformly generated at $r=r_a$ independently on θ and t. The remaining factors also may be assumed to be represented as a function of r alone. The temperature gradient must exist also in the direction of thickness, but it is enough to consider it on the average, because the thickness is so thin comparing with the radius. In this case the temperature is to be distributed concentrically and the state of temperature distribution is expressed as a function of radius r and time t.

From such a point of view we introduce a differential equation about the conduction of heat.

Considering the annular zone with infinitesimal width dr as shown in Fig. 1, the

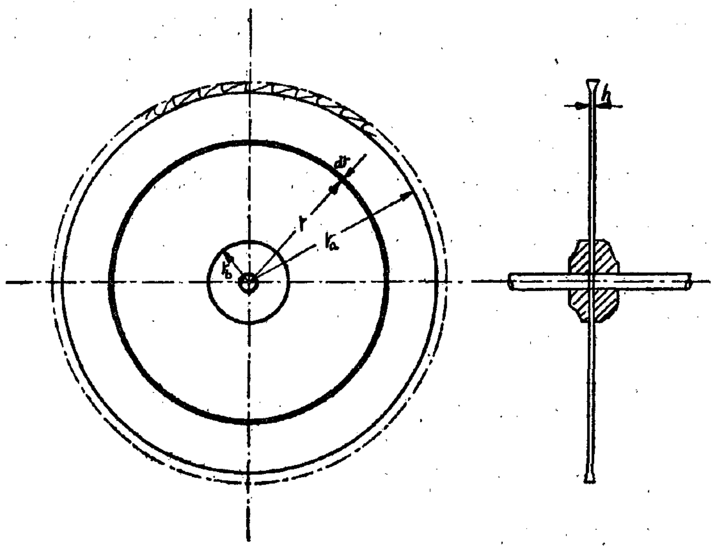


Fig. 1

quantity of heat that flows in to the positive direction of r through the inner wall of radius r in time dt is

$$-k\,\frac{\partial T}{\partial r}\,2\pi r\cdot h\cdot dt$$

k : thermal conductivity of steel

T : temperature of this place

On account of the oppositeness of the sign of temperature gradient $\frac{\partial T}{\partial r}$ and the one of direction of heat flow is given the negative sign. In the same way the quantity of heat that flows out to the positive direction of r through the outer wall of the radius $r+dr$ is

$$-\left\{ k \frac{\partial T}{\partial r} 2\pi r \cdot h + \frac{\partial}{\partial r} \left(k \frac{\partial T}{\partial r} 2\pi rh \right) dr \right\} dt$$

Thus the quantity of heat stored in this zone in time dt is

$$\frac{\partial}{\partial r} \left(k \frac{\partial T}{\partial r} 2\pi r \cdot h \right) dr \cdot dt$$

The heat dispersion into the air from the surface is to be proportional to the temperature difference $T - T_0$ (T_0 : the temperature of the air), when the temperature is not so high. So the quantity is expressed as follow,

$$2\alpha(T - T_0) 2\pi r \cdot dr \cdot dt$$

α : coefficient of heat transfer

Expressing by q the quantity of heat generated at the unit surface area by friction in unit time, the quantity of heat given to this zone by side friction in time dt is

$$2q \cdot 2\pi r \cdot dr \cdot dt$$

Supposing that consequently the temperature of this zone rises by dT, the following equation may be composed of the quantities described above.

$$c\rho \cdot 2\pi rh \, dr dT = \frac{\partial}{\partial r} \left(k \frac{\partial T}{\partial r} \cdot 2\pi rh \right) dr dt - 2\alpha(T - T_0) 2\pi r dr dt + 2q \cdot 2\pi r dr dt$$

$$\therefore \quad c\rho \frac{\partial T}{\partial t} = k \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) - \frac{2\alpha}{h} (T - T_0) + \frac{2q}{h} \quad \cdots\cdots (1)$$

c : specific heat of steel

ρ : density of steel

In the equation (1) expressing

$$\kappa = \frac{k}{c\rho}, \qquad a = \frac{2\alpha}{kh}, \qquad B = \frac{2q}{kh} \quad \cdots\cdots (2)$$

and transforming the vaviable by $\phi = T - T_0$, we obtain the following equation

$$\frac{1}{\kappa} \frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{\kappa} \frac{\partial \phi}{\partial r} - a\phi + B \quad \cdots\cdots (3)$$

This is a differential equation to determine the temperature distribution. When we know a and B or α and q as the functions of r, we may be able to solve this equation and determine the temperature distribution so as to be adapted to the bonndary and initial conditions, but not generally, even if a and B are found as the functions of r alone.

In this paper are treated the cases that both a and B are constant. More general cases with variable a and B will be treated in the successive paper.

Dispersion of heat from the side surface

As described above, we express the quantity of heat dispersing from unit area of the side surface in unit time by

$$\alpha(T-T_0)$$

where T_0 is the temperature of the air and may be taken for constant, and α is the coefficient of heat transfer, the value of which is fixed according to the properties of the fluid and the state of its motion.

The value of α generally exists within the extent of $10\sim500$ $kcal/m^2h^\circ C$ for the air in motion and $3\sim30$ $kcal/m^2h^\circ C$ in rest.

Let us consider the value of α in the occasion of circular sawblade. It may be considered to be determined only by the state of motion of the air on the surface of sawblde. But the actual state of motion is so much complicated due to the existence of machine table, wood being sawed and so on, that it is beyond all conjectures. As a case of simplicity let us consider a circular disk which rotates in the air with the angular velocity ω. Then the motion should be stationary and have a turbulent boundary layer.

Generally we can write by Re Reynolds' number, Nu Nusselt's number, σ Prandtl's number and k_H nondimentional heat transfer coefficient

$$Nu = k_H \sigma Re$$

$$Nu = \frac{\alpha r}{k_0}, \qquad Re = \frac{\omega r^2}{\nu_0}, \qquad \sigma = \frac{c_{q0}\rho_0\nu_0}{k_0}, \qquad k_H = \frac{\alpha}{c_{q0}\rho_0 r\omega}$$

ρ_0 : density of the air

k_0 : thermal conductivity of the air

c_{q0} : specific heat at constant pressure of the air

ν_0 : coefficient of kinematic viscosity of the air

On the other hand, by T. v. Kármán the following formulae are given, where c_f is the coefficient of the local surface friction in the turbulent boundary layer on the rotating disk

$$c_f = 0.0530\, Re^{-0.2} \qquad \cdots\cdots (5)^{4)}$$

and on the flat plate

$$\frac{1}{k_H} = \frac{g(\sigma)}{\sqrt{c_f}} + \frac{2}{c_f}, \qquad g(\sigma) = 5\left[(\sigma-1)+ln\{1+\frac{5}{6}(\sigma-1)\}\right] \cdots(6)^{5)}$$

Assuming that the relation (6) on the flat plate is able to exist on the rotating disk and taking $k_0=0.0220\ kcal/mh^\circ C$, $\nu_0=1.54\times10^{-5}\ m^2/s$, $\sigma=0.71$ for the air, we reach the next results from the equations (4), (5) and (6)

$$k_H \approx 0.029Re^{-0.2}$$

$$Nu=0.021Re^{0.8}$$

$$\therefore \quad \alpha=2.6\omega^{0.8}r^{0.6} \cdots\cdots(7)$$

This shows that the coefficient of heat transfer of the rotating disk in the air is proportional to $r^{0.6}$ and $\omega^{0.8}$.

As mentioned above, it is very difficult to presume the functional formula of α or a in our case, but in the sense of average it may be presumed as one of r alone and after the formula (7) as follows;

$$a=a'\omega^{0.8}r^{0.6} \cdots\cdots(8)$$

But for the constant value of a' we have now no ground to be able to presume it. Expecting another study about this problem in the future, in this paper we would treat it as a constant.

Calorification at the side surfaces

The quantity of heat generated from friction at the side surface is remarkably influnced by the conditions of the sawblade and sawing. It can be hardly estimated in actual sawing state, but it may be allowed to consider that on the average it is a function of r alone and that it is proportional to the products of the pressure of friction that may be assumed roughly proportional to r and l the length passing through the wood. In Fig. 2

$$l = 2r\left(\sin^{-1}\frac{M+r_c}{r} - \sin^{-1}\frac{r_c}{r}\right) \qquad \text{for } r_a \geq r \geq r_c+M.$$

$$l = 2r\cos^{-1}\frac{r_c}{r} \qquad \text{for } r_c+M \geq r \geq r_c$$

Thus the quantity of the heat q may be described as follows;

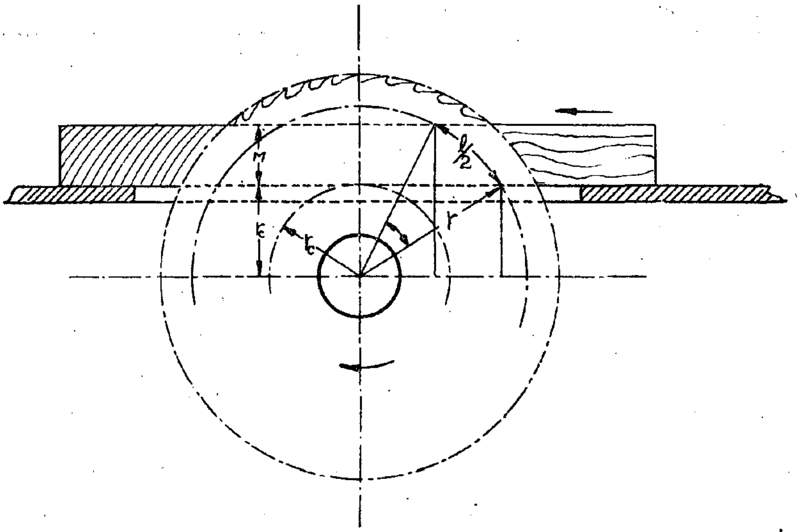


Fig. 2.

$$\left.\begin{aligned} q &= mr^2\left(\sin^{-1}\frac{M+r_c}{r} - \sin^{-1}\frac{r_c}{r}\right) && \text{for } r_a \geq r \geq M+r_c \\ q &= mr^2\cos^{-1}\frac{r_c}{r} && \text{for } M+r_c \geq r \geq r_c \\ q &= 0 && \text{for } r_c \geq r \geq 0 \end{aligned}\right\} \cdots\cdots\cdots(9)$$

where m is the constant, the value of which now cannot be presumed in whatever manner and even if q is expressed as (9), we could hardly solve the equation (3). We would treat it to be a constant in this paper and expect for more exact treatment in the another study about this problem.

Initial and boundary conditions

Until the commencement of sawing the blade only rotates in the air, so the temperature of the blade is to be uniformly equal to that of the air, neglecting the frictional resistance to the air. Thus the initial condition is ordinary

$$\phi_{t=0}=0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (10)$$

In the next place, when we represent graphically in Fig. 3 (a) the relations of the immediate temperature after each sawing (at measurement) at $r=r_a$ and the number of times of sawing from the experimental results[2] about sawing of SUGI (*Cryptomeria Japonica D. Don*) and NARA (*Quercus crispula Blume*), we find the relation linear and the number may be considered to be proportional to the elapsed time in sawing. Thus as shown in Fig. 3 (b) the boundary condition at the circumference is immediately

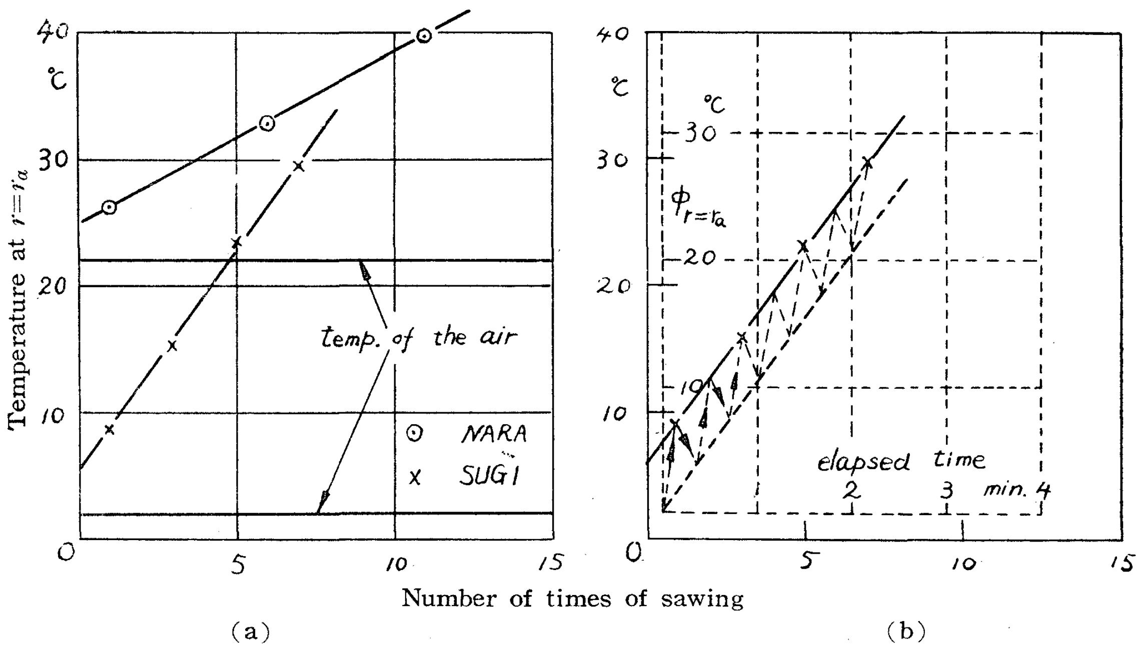


Fig. 3.

after sawing (as shown by full line)

$$\phi_{r=r_a}=Ct+D \quad\cdots\cdots(11)$$

where C and D are constants according to the conditions of sawing; and immediately before sawing (as shown by dotted line)

$$\phi_{r=r_a}=Ct \quad\cdots\cdots(11)_1$$

Some solutions by Carslaw's method[6] with Laplace-transformation

We multiply the both sides of the equation (3) by e^{-pt} (p : undecided coefficient) and integrate it with regard to t, taking 0 and ∞ as the limits of the integration.

$$\frac{1}{\kappa}\int_0^\infty e^{-pt}\frac{\partial\phi}{\partial t}dt=\int_0^\infty e^{-pt}\left\{\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}-a(r)\phi+B(r)\right\}dt \quad\cdots\cdots(12)$$

Applying the following Laplace-transformation to the equation (12),

$$\Phi(r)=\int_0^\infty e^{-pt}\phi(r,t)dt \quad\cdots\cdots(13)$$

we obtain the equation

$$\frac{d^2\Phi}{dr^2}+\frac{1}{r}\frac{d\Phi}{dr}-\{a(r)+p/\kappa\}\Phi+B(r)/p=0 \quad\cdots\cdots(14)$$

When we solve the equation (14) and make the Φ satisfy the conditions, we can express the solution $\phi(r,t)$ as follows,

$$\phi(r,t) = \frac{1}{2\pi i}\int_{g-i\infty}^{g+i\infty} e^{\lambda t}\, \Phi(r,\lambda)d\lambda, \qquad g>0 \quad \cdots\cdots(15)$$

by the table of Laplace-transformation or the inversion formula, that is, a function $f(t)$ which satisfy the Laplace-transformation

$$F(p) = \int_0^\infty e^{-pt} f(t)dt$$

is expressed as follows :

$$f(t) = \frac{1}{2\pi i}\int_{g-i\infty}^{g+i\infty} e^{\lambda t} F(t)dt, \qquad g>0.$$

If we could complete the complex integral (15), the solution of the equation (3) would be perfectly obtained.

Thus to solve the equation (3) resolves itself into solving the linear differential equation of second order (14) and the completion of the complex integration (15).

To succeed in these is not general. For the present paper, as above mentioned, some cases of a and B being constants are treated.

The case I : $a=0, \quad B=0$

where there is no dispersion and no calorification at the side surfaces.

This case coincides with that of heat conduction of the infinite cylinder which has been already known under the same conditions[7]. Similarly the solution is

$$\phi(r,t) = Ct - \frac{1}{4}\frac{C}{\kappa}(r_a{}^2 - r^2) + \frac{2C}{\kappa}\frac{1}{r_a}\sum_{n=1}^{\infty}\frac{J_0(\beta_n r)}{J_1(\beta_n r_a)\beta_n{}^3}\, e^{-\beta_n{}^2\kappa t} \quad \cdots\cdots(16)$$

β_n : the nth root of $J_0(\beta r_a)=0$

J_n : Bessel function of the first kind with the nth order

The case II : $a=a_0, \quad B=0$

where the dispersion is uniform and no calorification occurs at the side surfaces.

In this case the equation (14) reduces to

$$\frac{d^2\Phi}{dr^2} + \frac{1}{r}\frac{d\Phi}{dr} - (a_0 + p/\kappa)\Phi = 0$$

The solution of this equation is

$$\Phi(r) = \frac{C}{p^2}\frac{I_0(\sqrt{a_0+p/\kappa}\, r)}{I_0(\sqrt{a_0+p/\kappa}\, r_a)}$$

Similarly in the case I

$$\phi(r,t) = Ct\,\frac{I_0(\sqrt{a_0}\, r)}{I_0(\sqrt{a_0}\, r_a)} - \frac{C}{2\kappa\sqrt{a_0}}\left[\frac{r_a I_1(\sqrt{a_0}r_a)I_0(\sqrt{a_0}r) - rI_1(\sqrt{a_0}r)I_0(\sqrt{a_0}r_a)}{I_0^2(\sqrt{a_0}r_a)}\right]$$

$$+\frac{2C}{\kappa r_a}\sum_{n=1}^{\infty}\frac{\beta_n}{(a_0+\beta_n^2)^2}\frac{J_0(\beta_n r)}{J_1(\beta_n r_a)}e^{-(a_0+\beta_n^2)\kappa t} \quad \cdots\cdots (17)$$

β_n : the nth root of $J_0(\beta r_a)=0$

I_n : modified Bessel function of the first kind with the nth order.

The case III : $a=a_0$, $B=B_0'$ for $r_a \geq r \geq r_{a1}$

$a=a_0$, $B=B_0''$ for $r_{a1} \geq r \geq r_{a2}$

$a=a_0$, $B=0$ for $r_{a2} \geq r \geq 0$

In this case let us consider the problem, dividing the region of variable r into the three parts $r_a \geq r \geq r_{a1}$, $r_{a1} \geq r \geq r_{a2}$ and $r_{a2} \geq r \geq 0$.

【1】 For the region $r_a \geq r \geq r_{a1}$, taking $a=a_0$, $B=B_0'$ and $\phi=\phi_1$

$$\frac{1}{\kappa}\frac{\partial \phi_1}{\partial t} = \frac{\partial^2 \phi_1}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_1}{\partial r} - a_0\phi_1 + B_0', \quad \cdots\cdots (3)_1$$

$$(\phi_1)_{t=0}=0, \quad \cdots\cdots (10)_1$$

$$(\phi_1)_{r=r_a} = Ct. \quad \cdots\cdots (11)_1$$

【2】 For the region $r_{a1} \geq r \geq r_{a2}$, taking $a=a_0$, $B=B_0''$ and $\phi=\phi_2$

$$\frac{1}{\kappa}\frac{\partial \phi_2}{\partial t} = \frac{\partial^2 \phi_2}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_2}{\partial r} - a_0\phi_2 + B_0'' \quad \cdots\cdots (3)_2$$

$$(\phi_2)_{t=0}=0 \quad \cdots\cdots (10)_2$$

Further at the circle $r=r_{a1}$

$$(\phi_1)_{r=r_{a_1}} = (\phi_2)_{r=r_{a_1}} \quad \cdots\cdots (18)_1$$

$$\left(\frac{\partial \phi_1}{\partial r}\right)_{r=r_{a_1}} = \left(\frac{\partial \phi_2}{\partial r}\right)_{r=r_{a_1}} \quad \cdots\cdots (19)_1$$

【3】 For the region $r_{a2} \geq r \geq 0$, taking $a=a_0$, $B=0$ and $\phi=\phi_3$

$$\frac{1}{\kappa}\frac{\partial \phi_3}{\partial t} = \frac{\partial^2 \phi_3}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_3}{\partial r} - a_0\phi_3 \quad \cdots\cdots (3)_3$$

$$(\phi_3)_{t=0}=0 \quad \cdots\cdots (10)_3$$

Further at the circle $r=r_{a2}$

$$(\phi_2)_{r=r_{a2}}=(\phi_3)_{r=r_{a2}} \quad \cdots\cdots (18)_2$$

$$\left(\frac{\partial\phi_2}{\partial r}\right)_{r=r_{a2}}=\left(\frac{\partial\phi_3}{\partial r}\right)_{r=r_{a2}} \quad \cdots\cdots (19)_2$$

Solving the equation $(3)_1$, $(3)_2$ and $(3)_3$ with Laplace-transformation similarly in the case I and II, we obtain the Φ_1, Φ_2 and Φ_3 as follows, corresponding to the Φ of equation (4)

$$\Phi_1 = A_1 I_0(\sqrt{a_0+p/\kappa}\, r) + B_1 K_0(\sqrt{a_0+p/\kappa}\, r) + \frac{B_0'}{p(a_0+p/\kappa)} \quad \cdots\cdots (20)$$

$$\Phi_2 = A_2 I_0(\sqrt{a_0+p/\kappa}\, r) + B_2 K_0(\sqrt{a_0+p/\kappa}\, r) + \frac{B_0''}{p(a_0+p/\kappa)} \quad \cdots\cdots (21)$$

$$\Phi_3 = A_3 I_0(\sqrt{a_0+p/\kappa}\, r) \quad \cdots\cdots (22)$$

where I_0 is a modified Bessel function of the first kind, K_0 a modified Bessel function of the second kind and A_1, A_2, A_3, B_1, B_2 are arbitrary constants.

Determining the arbitrary constants A_1, A_2, A_3, B_1 and B_2 so as to satisfy the boundary conditions $(11)_1$, $(18)_1$, $(18)_2$, $(19)_1$, and $(19)_2$ and employing the Lommel's formula,

$$I_n(x)K_{n+1}(x)+K_n(x)I_{n+1}(x) = \frac{1}{x}$$

we obtain, substituting $\sqrt{a_0+p/\kappa}$ for ε

$$\begin{aligned}\Phi_1 = & \frac{C}{p^2}\frac{I_0(\varepsilon r)}{I_0(\varepsilon r_a)} - \frac{B_0'}{p\varepsilon^2}\left\{\frac{I_0(\varepsilon r)}{I_0(\varepsilon r_a)} - 1\right\} \\ & + \frac{B_0'-B_0''}{p\varepsilon} r_{a1}\{I_0(\varepsilon r)K_0(\varepsilon r_a) - K_0(\varepsilon r)I_0(\varepsilon r_a)\}\frac{I_1(\varepsilon r_{a1})}{I_0(\varepsilon r_a)} \\ & + \frac{B_0''}{p\varepsilon} r_{a2}\{I_0(\varepsilon r)K_0(\varepsilon r_a) - K_0(\varepsilon r)I_0(\varepsilon r_a)\}\frac{I_1(\varepsilon r_{a2})}{I_0(\varepsilon r_a)} \quad \cdots\cdots (20)'\end{aligned}$$

$$\begin{aligned}\Phi_2 = & \frac{C}{p^2}\frac{I_0(\varepsilon r)}{I_0(\varepsilon r_a)} - \frac{B_0'}{p\varepsilon^2}\frac{I_0(\varepsilon r)}{I_0(\varepsilon r_a)} \\ & + \frac{B_0'-B_0''}{p\varepsilon} r_{a1}\{I_0(\varepsilon r_a)K_1(\varepsilon r_{a1}) + K_0(\varepsilon r_a)I_1(\varepsilon r_{a1})\}\frac{I_0(\varepsilon r)}{I_0(\varepsilon r_a)} \\ & + \frac{B_0''}{p\varepsilon} r_{a2}\{I_0(\varepsilon r)K_0(\varepsilon r_a) - K_0(\varepsilon r)I_0(\varepsilon r_a)\}\frac{I_1(\varepsilon r)_{a2}}{I_0(\varepsilon r_a)} + \frac{B_0''}{p\varepsilon^2} \quad \cdots\cdots (21)'\end{aligned}$$

$$\Phi_3 = \frac{C}{p^2}\frac{I_0(\varepsilon r)}{I_0(\varepsilon r_a)} - \frac{B_0'}{p\varepsilon^2}\frac{I_0(\varepsilon r)}{I_0(\varepsilon r_a)} + \frac{B_0'-B_0''}{p\varepsilon} r_{a1}\{I_0(\varepsilon r_a)K_1(\varepsilon r_{a1})$$

$$+ K_0(\varepsilon r_a) I_1(\varepsilon r_{d1}) \} \frac{I_0(\varepsilon r)}{I_0(\varepsilon r_a)} + \frac{B_0''}{p\varepsilon} r_{d2} \{ I_0(\varepsilon r_a) K_1(\varepsilon r_{d2})$$

$$+ K_0(\varepsilon r_a) I_1(\varepsilon r_{d1}) \} \frac{I_0(\varepsilon r)}{I_0(\varepsilon r_a)} \cdots\cdots (22)'$$

Then calculating the complex integral (15) answering to above Φ_1, Φ_2 and Φ_3, the solution $\phi(r, t)$ to be required have been obtained as follows,

$$\phi_1(r_1 t) = C\Big[\frac{I_0(\sqrt{a_0}r)}{I_0(\sqrt{a_0}r_a)} t + \frac{1}{2\kappa\sqrt{a_0}} \frac{r I_0(\sqrt{a_0}r_a) I_1(\sqrt{a_0}r) - r_a I_0(\sqrt{a_0}r) I_0(\sqrt{a_0}r_a)}{I_0^2(\sqrt{a_0}r_a)}$$

$$+ \frac{2}{\kappa r_a} \sum_{n=1}^{\infty} \frac{\beta_n J_0(\beta_n r)}{(a_0+\beta_n^2)^2 J_1(\beta_n r_a)} e^{-\kappa(a_0+\beta_n^2)t} \Big] + \frac{B_0'}{a_0} \Big[1 - \frac{I_0(\sqrt{a_0}r)}{I_0(\sqrt{a_0}r_a)}$$

$$+ \sqrt{a_0} r_{d1} \frac{I_1(\sqrt{a_0}r_{d1})}{I_0(\sqrt{a_0}r_a)} \{ I_0(\sqrt{a_0}r) K_0(\sqrt{a_0}r_a) - I_0(\sqrt{a_0}r_a) K_0(\sqrt{a_0}r) \}$$

$$- \frac{\pi a_0}{r_a} \sum_{n=1}^{\infty} \frac{\frac{2}{\pi\beta_n} - r_{d1} Y_0(\beta_n r_a) J_1(\beta_n r_{d1})}{(a_0+\beta_n^2) J_1(\beta_n r_a)} J_0(\beta_n r) e^{-\kappa(a_0+\beta_n^2)t} \Big]$$

$$+ \frac{B_0''}{a_0} \Big[\{ r_{d2} I_1(\sqrt{a_0}r_{d2}) - r_{d1}(I_1\sqrt{a_0}r_{d1}) \} \{ I_0(\sqrt{a_0}r) K_0(\sqrt{a_0}r_a) - I_0(\sqrt{a_0}r_a)$$

$$\times K_0(\sqrt{a_0}r) \} \frac{\sqrt{a_0}}{I_0(\sqrt{a_0}r_a)} - \frac{\pi a_0}{r_a} \sum_{n=1}^{\infty} \frac{Y_0(\beta_n r_a) J_0(\beta_n r) \{ r_{d1} J_1(\beta_n r_{d1}) - r_{d2} J_1(\beta_n r_{d2}) \}}{(a_0+\beta_n^2) J_1(\beta_n r_a)}$$

$$\times e^{-\kappa(a_0+\beta_n^2)t} \Big] \cdots\cdots (23)$$

$$\phi_2(r, t) = C\Big[\frac{I_0(\sqrt{a_0}r)}{I_0(\sqrt{a_0}r_a)} t + \frac{1}{2\kappa\sqrt{a_0}} \frac{r I_0(\sqrt{a_0}r_a) I_1(\sqrt{a_0}r) - r_a I_0(\sqrt{a_0}r) I_1(\sqrt{a_0}r_a)}{I_0^2(\sqrt{a_0}r_a)}$$

$$+ \frac{2}{\kappa r_a} \sum_{n=1}^{\infty} \frac{\beta_n J_0(\beta_n r)}{(a_0+\beta_n^2)^2 J_1(\beta_n r_a)} e^{-\kappa(a_0+\beta_n^2)t} \Big]$$

$$+ \frac{B'_0}{a_0} \Big[- \frac{I_0(\sqrt{a_0}r)}{I_0(\sqrt{a_0}r_a)} + \sqrt{a_0}\, r_{d1} \frac{I_0(\sqrt{a_0}r)}{I_0(\sqrt{a_0}r_a)} \{ I_0(\sqrt{a_0}r_a) K_1(\sqrt{a_0}r_{d1})$$

$$+ I_1(\sqrt{a_0}r_{d1}) K_0(\sqrt{a_0}r) \} - \frac{\pi a_0}{r_a} \sum_{n=1}^{\infty} \frac{\frac{2}{\pi\beta_n} - r_{d1} Y_0(\beta_n r_a) J_1(\beta_n r_{d1})}{(a_0+\beta_n^2) J_1(\beta_n r_a)}$$

$$\times J_0(\beta_n r) e^{-\kappa(a_0+\beta_n^2)t} \Big] + \frac{B''_0}{a_0} \Big[1 - \sqrt{a_0} r_{d1} \{ I_0(\sqrt{a_0}r_a) K_1(\sqrt{a_0}r_{d1})$$

$$+ I_1(\sqrt{a_0}r_{d1}) K_0(\sqrt{a_0}r_a) \} \frac{I_0(\sqrt{a_0}r)}{I_0(\sqrt{a_0}r_a)} - \sqrt{a_0} r_{d2} \{ I_0(a_0 r) K_0(\sqrt{a_0}r_{d1})$$

$$- I_0(\sqrt{a_0}r_a) K_0(\sqrt{a_0}r) \} \frac{I_1(\sqrt{a_0}r_{d2})}{I_0(\sqrt{a_0}r_a)}$$

$$- \frac{\pi a_0}{r_a} \sum_{n=1}^{\infty} \frac{Y_0(\beta_n r_a) J_0(\beta_n r) \{ r_{d1} J_1(\beta_n r_{d1}) - r_{d2} J_1(\beta_n r_{d2}) \}}{(a_0+\beta_n^2) J_1(\beta_n r_a)} e^{-\kappa(a_0+\beta_n^2)t} \Big]$$

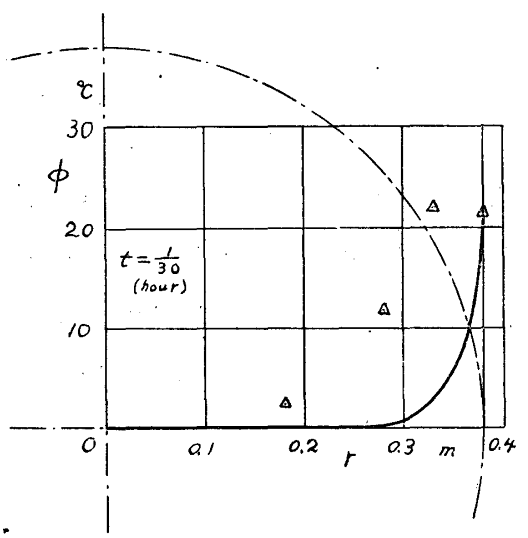
$$\cdots\cdots (24)$$

$$
\begin{aligned}
\phi_3(r,t) = C\Big[& \frac{I_0(\sqrt{a_0}\,r)}{I_0(\sqrt{a_0}\,r_a)}\,t + \frac{1}{2\kappa\sqrt{a_0}}\,\frac{r I_0(\sqrt{a_0}\,r_a)\, I_1(\sqrt{a_0}\,r) - r_a I_0(\sqrt{a}\,r) I_1(\sqrt{a_0}\,r_a)}{I_0^2(\sqrt{a_0}\,r_a)} \\
& + \frac{2}{\kappa r_a}\sum_{n=1}^{\infty}\frac{\beta_n\, J_0(\beta_n r)}{(a_0+\beta_n^2)^2 J_1(\beta_n r_a)}\, e^{-\kappa(a_0+\beta_n^2)t}\Big] + \frac{B_0'}{a_0}\Big[-\frac{I_0(\sqrt{a_0}\,r)}{I_0(\sqrt{a_0}\,r_a)} \\
& + \sqrt{a_0}\, r_{d1}\,\frac{I_0(\sqrt{a_0}\,r)}{I_0(\sqrt{a_0}\,r_a)}\,\{I_0(\sqrt{a_0}\,r_a)\, K_1(\sqrt{a_0}\,r_{d1}) + I_1(\sqrt{a_0}\,r_{d1})\, K_0(\sqrt{a_0}\,r)\} \\
& - \frac{\pi a_0}{r_a}\sum_{n=1}^{\infty}\frac{\frac{2}{\pi\beta_n} - r_{d1}\, Y_0(\beta_n r_a)\, J_1(\beta_n r_{d1})}{(a_0+\beta_n^2)\, J_1(\beta_n r_a)}\, I_0(\beta_n r)\, e^{-\kappa(a_0+\beta_n^2)t}\Big] \\
& + \frac{B_0''}{a_0}\Big[\Big\{ -\sqrt{a_0}\, r_{d1}\Big(I_0(\sqrt{a_0}\,r_a)\, K_1(\sqrt{a_0}\,r_{d1}) + I_1(\sqrt{a_0}\,r_{d1})\, K_0(\sqrt{a_0}\,r_a)\Big) \\
& + \sqrt{a_0}\, r_{d2}\Big(I_0(\sqrt{a_0}\,r_a) K_1(\sqrt{a_0}\,r_{d2}) + I_1(\sqrt{a_0}\,r_{d2})\, K_0(\sqrt{a_0}\,r_a)\Big)\Big\}\,\frac{I_0(\sqrt{a_0}\,r)}{I_0(\sqrt{a_0}\,r_a)} \\
& - \frac{\pi a_0}{r_a}\sum_{n=1}^{\infty}\frac{Y_0(\beta_n r_a)\, J_0(\beta_n r)\,\{r_{d1} J_1(\beta_n r_{d1}) - r_{d2} J_1(\beta_n r_{d2})\}}{(a+\beta_n^2)\, J_1(\beta_n r_a)}\, e^{-\kappa(a_0+\beta_n^2)t}
\end{aligned}
\qquad \cdots\cdots\cdots\cdots\cdots (25)
$$

Y_n : Bessel function of second kind with n th order

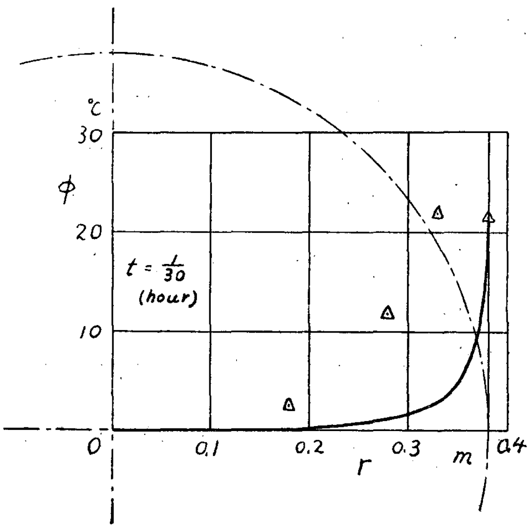
Results of calcultion and experiment

Let us calculate the values of ϕ at some steps of time t, taking the constants in the formulae of ϕ as undermentioned, and represent the relations between ϕ and r respectively in Fig. 4 - Case I, Fig. 5 - Case II and Fig. 6 - Case III.



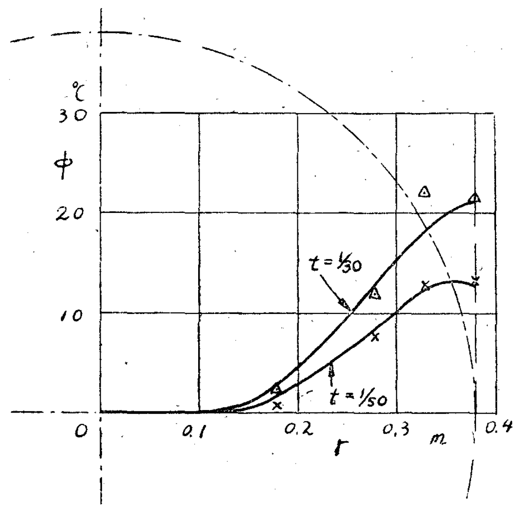
Case - I

Fig. 4



Case - II

Fig. 5



Case – III

Fig. 6

r_a = 0.38 m : radius of sawblade

r_c = 0.18 m : in Fig. 2

M = 0.18 m : height of wood sawed (in Fig. 2)

κ = $k/c\rho$ = 0.042 m^2/h : thermal diffusivity in sawblade

C = 630°C$/h$: C is the rising rate of temperature at $r = r_a$ as shown in the boundary condition $(11)_1$, and is influenced by every conditions of sawing. When we assume that it takes 20 seconds for one cyclic time of sawing of the SUGI wood — length about 2 m and height $M = 0.18m$ — C is 10.5°$C/min.$ or 630°C/h, because it rises by about 20°C for six times of sawing as shown in Fig. 3 (a)

k = 37 $kcal/mh\,°C$: thermal conductivity in sawblade

h = 1.65 mm : thickness of sawblade

$a_0 = 2\alpha_0/kh = 4031/m^2$; Assuming that α the coefficient of heat transfer from circular sawblade into the air is the half of the formula (7) in the case of rotating disk, we take α_0 as the mean value over the total surface area of sawblade and then $\alpha_0 = 24.6\ kcal/m^2h\,°C$. In this way we substitute the full line I for the dotted line I in Fig. 7.

$$\left.\begin{aligned} r_{a1} &= 0.30\,m \\ r_{a2} &= 0.20\,m \end{aligned}\right\}$$ we assume thus in Case III,

$$\left.\begin{aligned} B_0' &= 2q_0'/k\cdot h = 2.0 \times 10^4\,°C/m^2 \\ B_0'' &= 2q_0'/k\cdot h = 0.8 \times 10^4\,°C/m^2 \end{aligned}\right\}$$ Having now no ground to estimate the value of q_0, we take the value of a_0/B_0', a_0/B_0'' as much as the curves of

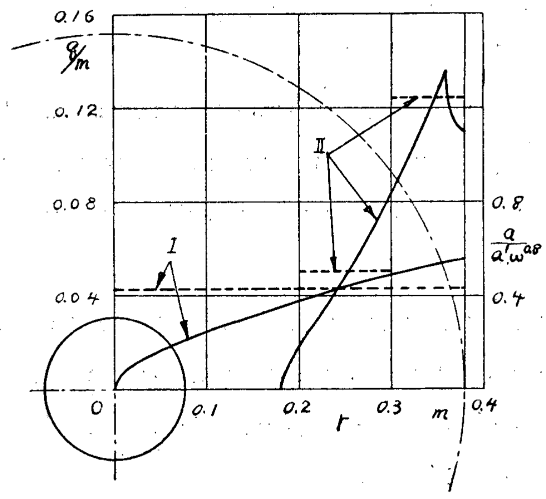


Fig. 7

calculated ϕ suit to the experimental results. We represent the relation between q and r in the equation (9) by the full line II in Fig. 7 and in this calculation substitute the full line II for the two dotted lines II, taking $a_0/B_0' = 1/50$, $a_0/B_0'' = 1/20$ and $r_{a1} = 0.30\ m$, $r_{a2} = 0.20\ m$ in the equation (23), (24) and (25).

In Fig. 8 we have shown the results[2] of experiment about SUGI wood and there

Conditions of sawing

$T_0 = 2°C$

$w = 112$ rad/s

$h = 1,65$ mm

width of kerf

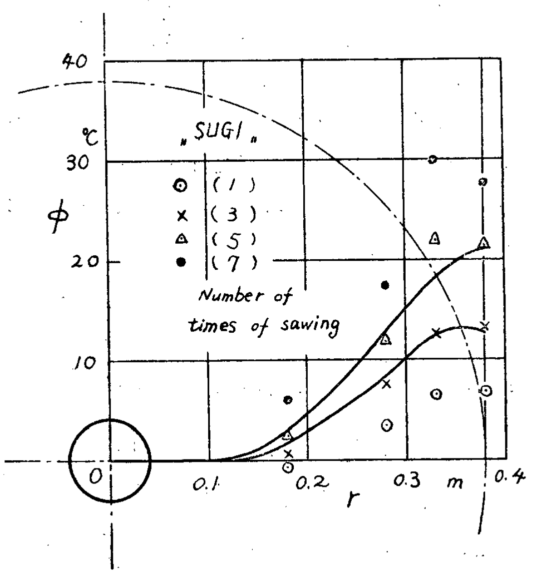
2,38 mm

$M = 180$ mm

$r_c = 180$ mm

$r_b = 76$ mm

(radius of flange)



Results of experiment about SUGI

Fig. 8

entered the curves calculated in the case III. The results of experiment are for the immediate value after sawing and that of calculation are for the immediate value before sawing. So as shown in Fig. 3 (b) the calculated temperature $r = r_a$ after 2 minutes corresponds to the experimental value after five times sawing.

Concluding Remarks

In this paper we only graphically represent both results of calculation and experiment and would not advance into the considerations about the practical problems, as the grounds of assumption about a and B are not sufficiently reliable and also the experimental results not satisfactory.

But we are assured that the factors influencing the temperature distribution have been analysed and the direction of the study on this problem has been found.

We expect further experimental or theoretical studies about those factors and the practical discussions in future.

Addition : This paper was read preliminary at the 64th meeting of Japanese Forestry Society on April 5. 1955. at Tokyo University.

Literature cited

1) H. SUGIHARA : Some problems about circular sawblade; Wood Research Review **4** P. 1/32 (1952). (In Japanese with English résumé).

2) Y. SAITO and T. NIGA : The Buckling of the circular saw; Wood Industry **9** No. 6 and No. 7 (1954) (In Japanese with English résumé).

3) J. R. Foyster : Modern mechanical saw practise P. 125 Crosby Lockwood and son L. T. D., (1953), London

4) T. V. Kármán : Über laminare und turbulente Reibung, Z. A. M. M. Bd. 1 Heft 4. (1921),

5) H. S. Carslaw : Conduction of heat Macmillan, (1921), London

5) K. KAWASHIMO : Conduction of heat, P. 207 Kawaide Shobō, (1940), Tokyo (in Japanese)

摘　　要

丸鋸の腰入れには先ず丸鋸刃の温度分布を知ることが第一であり，丸鋸の温度分布を決定する因子は　(1)外周に与えられる切削熱と摩擦熱　(2)側面よりの大気中えの熱の放散　(3)側面に於ける摩擦熱　の三つであるとして，これより熱伝導微分方程式を導き，これを粗雑ではあるが或仮定のもとに二三の場合について解き斎藤，仁賀氏の実験結果[2]と比較した。採用すべき仮定の検討とこれに関する研究及び腰入れの実際問題についての今後の研究を期している。